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ПРЕГЛЕДНИ ЧЛАНАК

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MODEL OF MANAGING MATHEMATICAL RESERVE OF LIFE INSURANCE

МОДЕЛ УПРАВЉАЊА МАТЕМАТИЧКОМ РЕЗЕРВОМ ЖИВОТНОГ ОСИГУРАЊА

Summary

Life insurance is a mutual guarantee of a large number of people with the same danger where the danger is random and can be measured and evaluated. The guarantee is represented by establishing the fund which is formed by money deposits made by endangered individuals who by doing that become members of the community of life insurance risk. These funds are used only to pay off the arranged amount to the member of the community when the insured accident occurs. A part of the financial means of the fund includes a mathematical reserve which serves as a collateral of future risks. These means are temporarily free means and are managed by the insurance company. The task of the insurance company is to keep the real value of the mathematical reserve as well as to increase its value. Therefore, the insurance company invests them, till the moment when these resources need to be used as collateral for the risk, in order to gain additional income. In order to achieve an efficient and economically efficient investment of the mathematical reserve, the original model has been designed. Designed and written model and its solution are the contribution of this paper. The key to the model provides a structure of the portfolio for investment of

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the mathematical reserve which ensures the required income from its investment with the minimum risk. Value added of the key to the model is the fact that it enables a post-optimal programming and simulation. This enabled a calculation of a sufficient number of investment portfolios' structures. Furthermore, this enabled a decision maker to use a high quality tool for managing risk of mathematical reserve investments as well as the overall risk of the life insurance.

Key words: life insurance, mathematical reserve, managing mathematical reserve, portfolio of investment, model of optimisation investment.

Резиме

Животно осигурање је узјамна гаранција великог броја људи исте угрожености, гдје је угроженост случајна и може се мјерити и процијенити. Гаранција се огледа у стварању новчаног фонда који се формира од уплата угрожених лица и којим чином уплатиоци постају чланови заједнице ризика животног осигурања. Ова уплаћена средства се користе искључиво за исплату уговореног износа члану заједнице када се оствари осигурани догађај. Финансијска средства фонда, једним дијелом, чини математичка резерва која служи за покриће будућих ризика. То су привремено слободна средства и њима управља осигуравател. Задатак осигуравателя је очување стварне вриједности математичке резерве, али, истовремено, и увећање њезине вриједности. Зато осигуравајуће друштво, у периоду до тренутка потребе кориштења ових средстава за покриће ризика, њих улаже са циљем остварења додатног прихода. За остварење економски ефикасног улагања математичке резерве, изграђен је оригинални стохастичко-математички модел. Изграђени и исписани модел и рјешење модела су допринос овог рада. Рјешење модела даје структуру портфеља улагања математичке резерве која осигурава потребан принос од њеног улагања уз најмањи ризик. Додатна вриједност модела је што омогућава постоптимално програмирање и симулацију, а што значи прорачун довољног броја структура портфеља улагања. Тиме је доносиоцима одлуке осигуран квалите-

тан алат за управљање ризицима улагања математичке резерве и цјелином ризика у животном осигурању.

Кључне ријечи: животно осигурање, математичка резерва, управљање математичком резервом, портфолио инвестирања, модел оптимизације инвестирања.

1. Introduction

Insurance includes mutual settling of numerous individuals who are endangered in the same way and where danger occurs accidentally and it can be evaluated. The point of insurance is payment of capital from the fund which is formed from deposits made by all individuals who participate in certain kind of community of risk, i.e. in the type of insurance. The task of insurance is therefore to distribute numerous dangers, which the insured individuals are exposed to, to all insured individuals and to pay the fee for the damage suffered or to pay the amount of money as it was arranged in the life insurance contract.

Life insurance is a special kind of insurance with its special features. It is a long-term and savings insurance of the certain amount of money used by the insured individual or by, in case of death, his/her inheritors. Therefore, a life insurance policy in the contemporary finance gains importance of a security and a high quality guarantee. Besides, the life insurance represents, due to its specific long-term quality, an essential source of investment with high return.

2. Risk in life insurance business activities

Risk is a source and basis of life insurance. Namely, life insurance is in its essence economic or to be more precise financial collateral for the risk of life, the probability of life. Mutual feature of all risks is the fact that the owner of the insurance policy receives the compensation in case the insured event happens in the amount previously arranged by the contract or the insurer receives the premiums with the calculated interests.

Hence, the aim of the insurance activities is to cover for the loss which is caused by the risk. In this type of insurance the individuals are insured and there are three conditions that need to be met: a) the event (the insured event) is prospective, b) uncertain and c) independent from the insurers will. The risk and damage have to be suitable for evaluation. Therefore, the risk in insurance represents a possibility of the accidental occurrence of the insured event, which causes the loss and its impact on the insured object in the future.

To establish the size of the risk precisely is very important for insurance i.e. it is relevant to measure the quantity of danger from the damaging event for the object of insurance. The size of the risk depends on many factors, firstly on a) the type of the danger, b) the value of the insurance and c) the length of the insurance.

The insurance companies predict the amount of payment and they charge the premium based on these predictions. The insurance companies face an additional risk which is the consequence of the quality of risk prediction i.e. of the prediction accuracy.

Risk can be measured and therefore it can be said that there is a higher and lower risk in a particular situation. These terms, higher and lower risk, are used in order to determine the measure of size of the potential compensation for the accidental event that will occur.

The probability of the loss and its size, if it really happens, contributes to the intensity of the reaction of one individual to the risk. Therefore, while measuring the risk, a range of potential losses has to be detected. On the other hand, if we compare two situations in which the amount paid for the loss is equal, the situation with the greater possibility for the loss brings along the higher risk. The interrelation between the size of a potential loss and the probability of that loss, while measuring the risk, represents the expected value.

Besides the risks of the probability of life, there are risks of the business environment as well, such as a) the risk of the interest rate calculated in the premium and b) the system risks that threaten the business activities of all financial institutions including the insurance companies.

3. Mathematical reserve

Mathematical reserve is a constituent part of the net deposit of life insurance. It is characteristic only for life insurance and it is calculated based on principles and methods of actuary mathematics. The calculation is based on the specific features of life insurance i.e. on the concept of equalisation of long-term risk. Namely, life insurance is achievable only by making an average deposit while risks increase over the time of insurance. Therefore it is necessary to create a reserve from the current and future payments for risk coverage which increases, and becomes larger even from the average payment, in the later period of life insurance.

Basic elements of risk in the life insurance, which need to be taken into consideration even in the mathematical calculation, are mortality and interest rate. Based on the assessment and calculation of the risk of life it is possible to determine an average insurance payment. If the average payment of life insurance is used, the cumulated means in the first period of the insurance will serve as coverage for the increased risks in the second period of the arranged insurance. Hence, the mathematical reserve represents a surplus of payment in the first period of the insurance or to be more precise it is accumulation of savings deposits invested to bear interests.

Life insurance is based on the principle: current value of deposits made by an insurer is equal to the current value of payment, liabilities which the insurer is obliged to cover to the user of the insurance policy. During the period of insurance, the risk that the insurer needs to cover is increasing, the payment of the insured amount of money is due and the insurer's liabilities are decreasing i.e. payment by instalments or payment is no longer made when the insurance is made through a single payment. Hence, the financial difference is created which constantly increases during the period of insurance, between the insurer's liabilities and the liabilities of the person insured. Therefore, the insurance company has to create reserves and the source of them lies in the deposits of the insured. This reserve, given in a calculation, is a difference between the insurer's capital liabilities and liabilities of the insured person at a certain moment on a certain day. It actually represents an amount of money required for covering liabilities to the user of insurance at the certain moment on a certain day.

3.1. Calculation of mathematical reserve

For all kinds and models of life insurance and for both types of payments, single or in instalments (recurring), temporary or lifelong, immediate or delayed, the same principle is applied as well as the general mathematical relation for calculation of the mathematical reserve. Therefore, in this paper we will analyse and provide a model for a mathematical reserve of a life insurance. This will be sufficient for creating a model for mathematical reserve and its calculation for any type of life insurance.

Hence, let us take an example of the insurance in case of death of K monetary units of a person who is x years old who will, starting from the day when the contract was concluded, pay an equal annual sum for life.

Real annual payments for death insurance coverage i.e. payments which include risk expressed in money value in a certain year, would, for a single monetary unit, equal the following:

$$\frac{M_x - M_{x+1}}{D_x}, \frac{M_{x+1} - M_{x+2}}{D_{x+1}}, \frac{M_{x+2} - M_{x+3}}{D_{x+2}}, \ldots, \frac{M_{x+s} - M_{x+s+1}}{D_{x+s}}$$

These payments maintain fast growth. They follow the actual risk of death and therefore they are called natural payments. Insurance model with natural payments is only a theoretical model and it is not acceptable or possible in actual life insurance activities.

In the real life of life insurance and business activities of the insurer, the insured make equal payments within the whole period of insurance. These payments for a monetary unit of the insured sum of the type of insurance we observe, is calculated by using the following relation

$$P(A_x) = \frac{M_x}{N_x}$$

With these payments the insured pays significantly larger amounts in the initial period (during the first years) than the natural payment.

The insurer does not need this extra amount paid to cover the risk of life i.e. to pay the due payments, therefore, this extra amount is kept as a reserve and will be used in the period when the same payment will not be sufficient for the payment of the insurer.

The life insurance paid through the equal payments enables the insured to gain the cover for the higher risk which is characteristic for the later years of life. In case the natural payment is made during the whole insurance period, the large payments would become due exactly in the time when the person insured is older, which is the period when the person insured has not got the opportunity to earn enough to be able to make such large payments. Simply put, the equal amounts of payments are adjusted to the objective possibilities of the insured and the rationalisation of the risk cover is achieved.

The equal payment consists of two parts: the first one which covers the currents risk and is called the risk of payment and the second one which is saved for the risk cover in the period when the payment is not sufficient to cover the risk and is called the savings payment[1]. The savings payment is calculated by subtracting the risk of payment from the total amount of payment.

The payment made by the insured is constant, relatively low and acceptable even in old age. A part of the payment is used by the insurance company to cover the death. That is the payment of the insured amount for the dead (the risk of payment) and the rest is kept as a reserve (the savings payment). The risk of payment increases as the time passes which is caused by the higher risk of death.

This method, specific for all types of life insurance, involves relatively low and constant payments and enables the life-long insurance. Actually, the point is that people save in young age for the old age[2].

The mathematical reserve forms the basis of the business activities of an insurance company. Calculating and managing it are the essential

[1] The savings premium is a part of the net premium which acts as the cover for the future liabilities of the insurer which then settles the risk cover during the whole insurance period. The risk of payment is a part of the here premium which acts as the cover for the current risk during the insurance period.

[2] Life insurance is a specific type of savings.

tasks necessary for the survival of the insurer on the insurance market. The mathematical reserve is made out of the rest of the payment after covering the current risk. Its source is, therefore, the savings payment which consists of savings payment with calculated interests. This means that the mathematical reserve in the particular moment (on the particular day) equals the total amount of all, until that day, due payments and savings payments with calculated interests.

The calculation of the mathematical reserve can be made by using two methods:

a) calculating deposits and payments in the last period – the retrospective method;
b) calculating the future deposits and payments – the prospective method.

Naturally, both ways generate the same result; the method cannot change the result.

a) The retrospective method

The mathematical reserve equals the value of deposits made till that day reduced by the value of payments made till that day. Therefore, it is necessary to establish the current value of all deposits and payments.

b) The prospective method

The calculation of the mathematical reserve can be made based on the future deposits and payments. Namely, we will rely on the principle: the mathematical reserve is supposed to cover all future liabilities (payments) of the insurer. The total of all of those liabilities equals the value of the future payments on the day the calculation is made reduced by the value of future deposits on the day the calculation is made. These values are gained by discounting of the nominal values of the future payments and deposits.

We will try to determine the mathematical reserve within the analysed model, which includes single life-long payments in the case of death with the life-long equal annual deposits, in the period t (years) after the beginning of the insurance with the value $t = 20$. The relation for the calculation of the mathematical reserve for the single currency unit insured is the following:

a) by using the retrospective method

$$ {}_{20}V_x = \frac{M_{30}}{N_{30}} \cdot \frac{N_{30} - N_{50}}{D_{30}} \cdot \frac{D_{30}}{D_{50}} - \frac{M_{30} - M_{50}}{D_{30}} \cdot \frac{D_{30}}{D_{50}} $$

which for 1 currency unit amounts to

$$ {}_{20}V_{30} = 0,2532918 $$

b) by using the prospective method

$$ {}_{20}V_x = \frac{M_{50}}{D_{50}} - \frac{M_{30}}{N_{30}} \cdot \frac{N_{50}}{D_{50}} $$

and for 1 currency unit o the insured amount equals

$$ {}_{20}V_{30} = 0,2532918 $$

The calculated value represents temporarily free financial means till the moment of covering the taken liabilities based on the arranged insurance.

The insurance company has to invest the mathematical reserve with calculated interests in securities, mortgage loans, deposits in banks and similar. It is important to emphasise that there are legal limitations to the structure of investments and prohibition of investments in speculations activities, trade and similar. A reason for this is minimising the investment risk to the lowest level. The mathematical reserve is actually a high quality asset committed by the insured to the insurer's care and management.

The mathematical reserve is a liability of the insurer's balance, which means that it represents a long-term liability of the insurance company for the payment of the arranged amounts of life insurance. The constant growth, absolute and relative and the elements of the mathematical reserve within balance sheet liabilities express higher temporarily free financial means, large liabilities towards the insured but at the same time larger investment means. These means represent the means of the insurance risk community and with economic and legal limitations they are managed by the insurance company. The methods used for managing these means have a direct impact on the quality of business activities of the insurance company. It is noticed in the quality of the model of investment portfolio of the mathematical reserve. Namely, every insurance company in a particular country has the same macroeconomic conditions as well as similar values of parameters of the calculation of the insurance net deposit. Professional analysis and notion of the importance of parameters as well as the quality of the investment portfolio of the mathematical reserve determine the success of every insurance company's business activities. Consequently, it is necessary to design high quality models for managing investment risks of the mathematical reserve means.

4. Managing mathematical reserve

Life insurance is a long-term investment of the insured, members of the risk community, in the insurance institution. This obliges the insurance company to create the portfolio of investment of the means of mathematical reserve which will achieve returns not less than the ones that are promised by the insurance contract. Besides investing mathematical reserve in order to gain returns, these means, at the same time, have to be at disposal for the expected payments of arranged insured amounts and for life coverage insurance. Furthermore, the insurer has the right to pay the life insurance to the amount of purchase value before its due time.

Such demands, which include risk and long-term savings deposits, require from the insurance companies to establish a stable mathematical reserve. The concept of managing the mathematical reserve as well as the capital, which derives from it and which is its constituent part, is of the utmost importance for the stability and survival of the insurance company. However, it should be added and emphasised that managing the mathematical reserve is not the exclusive right and obligation of the insurance company. The importance of managing the mathematical reserve is beyond the independence of the insurance company and its management. The investment of the mathematical reserve represents an important factor of the financial structure of one country because the insurance company acts as an institutional investor that with its investments on the financial markets allocates the assembled financial means of the citizens which are now present in the form of savings deposits. The stability of the financial system as well as the protection of the insurer is of the strategic interest of every country[3]. Therefore, the regulators, taking that into account, impose the legal obligation of professional managing the mathematical reserve to every insurance company which specialises in life insurance.

4.1. Risks of investing the mathematical reserve

Investing the mathematical reserve involves risks which from the point of view of a particular level of management of the insurance company can be: a) external and b) internal.

a) External risks are caused and determined by economic and social environment in which the insurance company does its business activities but cannot have much influence on. However, the insurer has to analyse them and determine the degree of its influence as well as include them in the model of optimisation of the investment portfolio. These risks are the consequence of the economic system and are called systemic risks.
The economic system consists of a range of regulations imposed by the government in order to control the economy. The contents of the economic policy represent instruments which ensure that the economy functions according to a particular system and set goals. The particular system and policy determine the environment and the position of every business activity which is a part of the system. Investing the temporarily free means of the life insurance, the mathematical reserve, belongs to the field of finance or to be more precise to the capital market. This market includes the common risk of all investments made and therefore forms a system risk. This risk cannot be changed on the particular market. It cannot decrease or increase because it is a consequence of the in-

[3] A special kind of additional limitations to investments of the mathematical reserve are of the great importance.

fluence factors which are determined by the particular economy and they derive from it[4].

b) The internal risk is specific for single investments in single security and it is called non-systemic risk. This group of risks is directly run by the insurance company. This risk is a result of the quality and changeability of business activities of an economic subject, the one that issues securities. Securities of a particular economic subject are the ones that bear risk. Investing the mathematical reserve in securities requires the right choice of securities and forming of investment portfolio i.e. analysing and measuring risks as well as achieving financial profit, return on investment. This brings us to the fact that managing risks of investment of mathematical reserve basically means designing the investment portfolio. In order to manage the mathematical reserve efficiently it is necessary and possible to design a model which includes only non-systemic risks with the possibility to optimise the profit of investing in risks. Therefore, there is a need to determine the system of investment portfolio and share of securities which ensure 1) the highest profit with risk or 2) the lowest risk for expected single returns generated from securities which form a part of the portfolio.

5. Model of optimisation of mathematical reserve investment

While working on the design of the model of optimisation of mathematical reserve investment, and referring to the research made so far and gained notions about life insurance, we would like to explain our task as follows.

[4] In developed countries there are significant opportunities for investments of the mathematical reserve either by the range or the quality of investments. The opportunities for investments include loans, bank deposits, government and corporation bonds, investment funds and similar. Such investments are not possible for the insurance companies in undeveloped market economies which also means in economies of countries in transition. Therefore, the system risk in these countries is much higher, changeable and it is difficult to measure it.

Insurance company has a certain amount of mathematical reserve at its disposal. These means can be invested in a particular number of securities which can gain profits, however they include risk as well. The total value of securities equals the sum of mathematical reserve. There are certain legal limitations for particular types of investments of these temporarily free means[5]. The aim would be to establish the integral parts of the total investment which will ensure certain profits on the means invested with the minimal investment risk.

In order to accomplish such an aim, it is necessary to design a model and find its solution[6]. While designing the model it is inevitable to establish the elements of the presented problem and carry out their quantification. The elements of the presented problem are given the following symbols:

- R - portfolio's profits made from investments;
- K - mathematical reserve means used for investments;
- W_i - nominal amount of securities *i*, $i = 1,2,...,n$
- n - number of securities which are included in the investment portfolio;
- *i* - the symbol for a security, $i = 1,2,...,n$;
- g_t - limitations of investment in *i* security;
- w_i, w_j - a share of securities *i* (or *j*) in the portfolio;
- R_i - profits made from securities' investments *i*;
- $E(R_i)$- expected profits made from investing securities *i*.

In order to measure the portfolio risk, the evaluated variations of the expected portfolio profits are considered. Mathematical calculation of that magnitude is the following:

$$s^2 = \sum_{i=1}^{n}\left(\sum_{j=1}^{n} w_i w_j s_i s_j\right)$$

[5] „Law of Insurance Companies in the Private Insurance“, article 11., 57., and 82., Official Journal of the Federation of Bosnia and Herzegovina, number 24/05.

[6] Harry M. Markowitz, *Mean-Variance Analysis in Portfolio Choice and Capital Markets*, Basil Blackwell Inc., Oxford 1987 and Harry M. Markowitz, *Portfolio Selection*, Basil Blackwell Inc., Oxford 1991.

$$s^2 = w_1 w_1 s_1 s_1 + w_1 w_2 s_1 s_2 + ... + w_2 w_1 s_2 s_1 + ... + w_i w_j s_i s_j + ... + w_n w_n s_n s_n$$

while

$$w_i = \frac{W_i}{K}$$

$$s_i = E\left(R_i\right) - R_i \quad i = 1,2,...,n$$

However, if $i = j$, then magnitude s_i, s_j represents the evaluated variation of the securities and if $i \neq j$ it represents the evaluated co-variation.

The given variation of the expected portfolio profits takes over the function of the aim Z whose, with the established limitation, conditioned minimum represents the lowest risk of portfolio investments. Limitations are established for the total amount of relative shares of securities which is as follows:

$$w_1 + w_2 + ... + w_i + ... + w_n = 1$$

as well as the legal limitations to the shares in the portfolio of certain securities

$$w_1 + w_2 + ... + w_r = g_i$$

and the prerequisite for the level of portfolio returns

$$E(R_1)w_1 + E(R_2)w_2 + ... + E(R_i)w_i + ... + E(R_n)w_n = R$$

We are now able to present the whole model of portfolio investment optimisation:

The values of the changeable $w_1, w_2, ..., w_n$ should be determined, which ensure the minimum value of the function

$$Z = w_1 w_1 s_1 s_1 + w_1 w_2 s_1 s_2 + ... + w_2$$

$$w_1 s_2 s_1 + ... + w_i w_j s_i s_j + ... + w_n w_n s_n s_n$$

with limitations

$$w_1 + w_2 + w_3 + ... + w_i + ... + w_n = 1$$

$$E(R_1)w_1 + E(R_2)w_2 + \ldots + E(R_i)w_i + \ldots + E(R_n)w_n = R$$

$$w_1 + w_2 + \ldots + w_r = g_t,\ \ g_t < 1 \text{ i } t = 1,2,\ldots,m$$

In order to find the solution, the Langrang function for determining conditional extreme will be used, which for the model presented is as follows:

$$L = w_1 w_1 s_1 s_1 + w_1 w_2 s_1 s_2 + \ldots + w_i w_j s_i s_j + \ldots + w_n w_n s_n s_n + \lambda_1 (w_1 + w_2 + \ldots w_n - 1) +$$
$$\lambda_2 \left[E(R_1)w_1 + \ldots + E(R_i)w_i + \ldots + E(R_n)w_n - R) \right] + \lambda_t (w_1 + w_2 + \ldots w_r - g_t)$$

When we calculate the partial deduction of the function *L* according to the changeable $w_i, \lambda_1, \lambda_2, \lambda_t$ and equalise it with the value 0, we will gain the system of $n + 2 + t$ linear equations:

$$\frac{\partial L}{\partial w_i} = 2s_i s_i w_i + 2s_i s_j w_j + \lambda_1 + \lambda_2 E(R_i) + \lambda_t = 0$$
$$\frac{\partial L}{\partial \lambda_1} = w_1 + w_2 + \ldots + w_n - 1 = 0$$
$$\frac{\partial L}{\partial \lambda_2} = E(R_1)w_1 + E(R_2)w_2 + \ldots + E(R_n)w_n - R = 0$$
$$\frac{\partial L}{\partial \lambda_t} = w_1 + w_2 + \ldots + w_r - g_t = 0$$

for $i = 1,2,\ldots,n$ whose solution provides *n* values w_1 which determine the shares of securities in the portfolio. Such contents of the portfolio ensure a set level of profits made from investments with the minimum investment risk.

Since $(0,0,\ldots,1,R,g_t)$ represents a vector of free factors of the equation system, the given system can be used as a reliable means for managing the portfolio system i.e. the risk of investment of mathematical reserve. Namely, changing the parameter *R* we gain various contents of the portfolio. Furthermore, if the changes in the legal limitations of investments in some securities happen, the value of the parameter g_t will change. By using such an equation system, the calculation of value of shares of securities in the portfolio in case of the new limitations is possible and relatively easy. Therefore, the above explained model of optimisation enables the parameter programming and simulation analysis. These features add to its quality and make it especially useful for managing risks of investment of mathematical reserve.

6. Conclusion

Life insurance is the mutual guarantee of a large number of individuals with the same jeopardy, where jeopardy represents something accidental and can be measured and estimated. The guarantee is seen in the accumulation of the capital fund which is formed from the deposits made by all the jeopardised individuals and by this act the depositors become members of the community of the life insurance. These means are used primarily for paying the arranged sum to the member of the community when the determined event of jeopardy happens. The jeopardy in the life insurance is represented in the risk of life and dying. This means that the essence of the life insurance lies in the risk of life of one person.

Within the business activities of providing the life insurance a part of the financial means is represented in the form of mathematical reserve whish is used for covering the future risks. These temporarily free means are managed by the insurer. His duty is to preserve its real value but at the same time to increase its value. Therefore, the insurance company invests the temporarily free financial means for covering risks till the moment they need to be used in order to gain additional profits.

In order to achieve economically efficient investment of temporarily free financial means, it is necessary to establish the model for managing these means. The established and presented model and its solution have met this requirement. The solution to the model provides the structure of the portfolio of mathematical reserve investment which ensures the profits set by its investment with the minimum risk. The additional value to the solution of the model is the fact that it enables the post-optimal programming and simulation. This has ensured the model and its solution which, with the minimum of investment risk, enables the calculation of the sufficient number of structures of investment portfolios. In this way, the decision-makers are also equipped with the high quality means for managing the risks of mathematical reserve investments and the overall risk in the life insurance from the moment of establishing the obligation relationship till the end of insurance.

Literatura

1. Andrijanić, I.; Klasić, K. *Tehnika osiguranja i reosiguranja*. Zagreb: Ekonomski fakultet, 2002.
2. Andrijašević, S.; Petranović, V. *Ekonomika osiguranja*. Zagreb: ALFA, 1999.
3. Andrijašević, S.; Račić-Žlibar, T. *Riječnik osiguranja*. Zagreb: Masmedija, 1997.
4. Andrijić, S. Matematička pričuva temeljna odrednica poslovanja osiguravajućeg društva.// *Banke*, br. 4. Sarajevo, 2001.
5. Benjamin, B.; Pollard, J. *The Analysis of Mortality and Other Actuarial Statistics*. Institute and faculty of Actuaries, 1997.
6. Bijelić, M. *Osiguranje*. Zagreb: Birotehnika, 1990.
7. Black, K.; Skipper, H. *Life Insurance*. *12 the ed*. Prentice-hall, Englewood cliffs, 1994.
8. Borch, K. *The Matematical Theory of Insurance*. Toronto; London: Lexington Books, 1974.
9. Carić, S. *Bankarski poslovi i hartije od vrednosti*. Beograd: Savremena administracija, 1998.
10. Chiang, A. C. *Osnovne metode matematičke ekonomije*. 3. izd. Zagreb: Mate, 1996.
11. Filipović, S.; Jirasek, V. *Finansijska i aktuarska matematika*. Sarajevo: Zavod za izdavanje udžbenika, 1972.
12. Foley, J. B. *Tržišta kapitala*. Zagreb: Mate, 1993.
13. Frančišković, I. *Osiguranje u tržišnom gospodarstvu*. Zagreb: Croatia osiguranje, 1992.
14. Gogala, Z. *Neke značajke i postavke teorije selekcije portfolija.* Zagreb: Financijska praksa, 1996.

15. Haberman S.; Booth, P.; Chadburn, R.; Cooper, R.; James, D. *Modern Actuarial Theory and Practice*. London: Champan & Hall, 1998.
16. Ivanović, I. *Teorija verovatnoće*. Beograd: Naučna knjiga, 1977.
17. Livingston, M. *Mony & Capital Markets-Financial Instruments & Their Uses*. New York: Prentice Hall, Englewood Cliffs, 1990.
18. Krčmar, M. *Finansijka matematika i metode investicionog odlučivanja.* Sarajevo: Kemigrafika, 2002.
19. Markowitz, M.H. *Portfolio selection*. Oxford: Basil Blackwell Inc., 1991.
20. Markowitz, M.H. *Mean-Variance Analysis in Porfolio Choice and Capital Markets.* Oxford: Basil Blackwell Inc., 1987.
21. Pešić, M. Ulaganje sredstava životnog osiguranja bitna sastavnica marketing miksa osiguravajućeg društva.// *Ekonomski pregled,* 7-8/56, Zagreb, 2005.
22. Pojatina, D. *Tržište kapitala*. Split: Ekonomski fakultet, 2000.
23. Popović, M. S. *Portfolio analiza*. Podgorica: Ekonomski fakultet, 2000.
24. Prohaska, Z. *Analiza vrijednosnih papira*. Zagreb: Informator, 1996.
25. Sanjković, M. *Matematika osiguranja života*. Zagreb: Croatia osiguranje, 1974.
26. Vaughan, V. T.; Vaughan, T. *Osnove osiguranja*: *upravljanje rizicima*. Zagreb: Mate, 2000.
27. Vranić, V.; Martić, Lj. *Matematika za ekonomiste*. Zagreb: Školska knjiga, 1967.